MATHEMATICAL MODEL FOR A BACKPACK LOAD CARRIAGE

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ABSTRACT

A mathematical model for the analysis of human motion during walking while carrying a backpack is presented in this paper. The model consists of a human torso carrying a backpack in a sagittal plane. The objective of this paper is to develop a model for a backpack load carriage. A Newton-Euler equation is applied to describe the backpack kinematics. Using the inverse dynamics method, the developed model makes it possible to calculate the forces and moments for variation of parameters

INTRODUCTION

Backpacks are a widely accepted form of load carriage not only by soldiers and hikers but also by school children. Studies have shown that carrying heavy backpacks may lead to changes in trunk posture and muscle activity (Goh et al., 1998; Li and Hong, 2001; Hong and Cheung, 2003; Shasmin et al., 2007) especially for children who are still experiencing significant growth and motor development. While walking or running with a backpack the forces exerted on the body which is caused by the interaction of the backpack and the trunk, lead to relative motion of the pack with respect to the trunk. Therefore in order to determine these forces, a mathematical model is developed to analyze the backpack load carriage.

Biomechanical analysis of the backpack load carriage have been used in many studies, however the construction of the biomechanical models which requires the use of a formulation to describe the dynamics equation of motion only being studied by few researchers. Fossaic et al. (2008) proposed a simple characteristic of the mechanical properties of a backpack suspension system using various degrees of stiffness. Ren et al. (2005) studied the biomechanical effect of load carriage dynamics during human walking using a non-linear model to investigate the biomechanical effects of different backpack suspension characteristics. A static biomechanical model of load carriage has been developed by Pelot et al. (2000). By attaching the suspension system components to the best location on the model, the model incorporates the primary forces at the shoulder and waist belt. However, most of these biomechanical analyses of the backpack load carriage were studied on adults and soldiers and not on primary school children. Thus, this paper aims to develop a mathematical model to calculate the forces and moments acting on the body during walking while carrying a backpack.

BIOMECHANICAL MODEL

This paper presents a mathematical model of a human torso carrying a backpack in a sagittal plane. A Newton-Euler equation is applied to describe the backpack kinematics. Using an inverse dynamics, the model is developed to describe the backpack's dynamic response to trunk motion.

Nomenclature for the free body diagram

F_{xp}, F_{yp}	- normal and tangential backpack interface forces acting on the pack center of mass
M_{zp}	- moment about the pack's center of mass
$\ddot{x}_p, \ddot{y}_p$	- normal and tangential accelerations of the backpack center of mass
m_p	- mass of the backpack
I_p	- moment inertia of the pack
α_p	- angular acceleration of the pack
$\ddot{x}_t, \ddot{y}_t$	- horizontal and vertical accelerations of the trunk mass center
$\theta_t, \omega_t, \alpha_t$	- angular displacement, velocity and acceleration of the trunk
dx, dy	- normal and tangential positions of the pack mass center relative to the torso mass center
u, $\dot{u}$	- displacement and velocity of the pack suspension system

In this study, two moving coordinates systems, the backpack coordinate system $O_t x_t y_t$ and the trunk system $O_p x_p y_p$ are show in Figure 1. For simplification, the internal deformation of the backpack is neglected and the backpack is modeled as a rigid body that moves with the trunk. The shoulder straps prevent the rotation and translation of the backpack relative to the trunk along the x_p direction. Using the Newton-Euler equation, the forces and moments that involved in the backpack and trunk system can be evaluated. The equation is applied to describe the backpack kinematics in the horizontal and vertical components. The x-component of forces acting on the backpack is given by

$$\sum F = m_p \ddot{x}_p = F_{xp} + mg\sin\theta_t \tag{1}$$

and the y-component of forces acting on the backpack is

$$\sum F = m_p \ddot{y}_p = F_{yp} - mg\cos\theta_t \tag{2}$$

The pack moment can be obtained from

$$M_{zp} = I_p\alpha_p - M_{xp} - M_{yp} = I_p\alpha_p - F_{xp}(dy+u) - F_{yp}\cdot dx \tag{3}$$

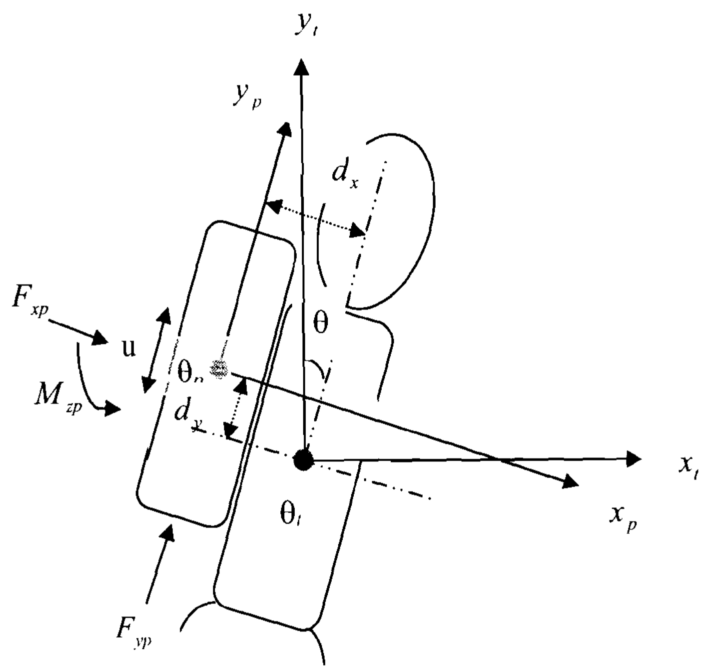

FIGURE 1 A Free Body Diagram of a Rigid Body Model of a Backpack and a Trunk

The position of the pack center of mass relative to the trunk in the x and y components can be written as

$$x_p = x_t\cos\theta - y_t\sin\theta \tag{4}$$

$$y_p = x_t\sin\theta + y_t\cos\theta \tag{5}$$

Differentiation of equations (4) and (5) with respect to θ, yield

$$\begin{aligned}\dot{x}_p &= \dot{x}_t\cdot\cos\theta + x_t(-\sin\theta)\omega_t - [\dot{y}_t\cdot\sin\theta + y_t\cdot\cos\theta\cdot\omega_t] \\ &= \dot{x}_t\cdot\cos\theta - \dot{y}_t\cdot\sin\theta - \omega_t(x_t\cdot\sin\theta + y_t\cdot\cos\theta)\end{aligned} \tag{6}$$

$$\begin{aligned}\dot{y}_p &= \dot{x}_t\sin\theta + x_t\cos\theta\cdot\omega_t + \dot{y}_t\cos\theta + y_t(-\sin\theta)\cdot\omega_t \\ &= \dot{x}_t\sin\theta + \dot{y}_t\cos\theta + \omega_t[x_t\cos\theta - y_t\sin\theta]\end{aligned} \tag{7}$$

The second differentiation of x and y components gives

$$\ddot{x}_p = \ddot{x}_t \cos\theta - \ddot{y}_t \sin\theta - 2\omega_t(\dot{x}_t \cdot \sin\theta + \dot{y}_t \cdot \cos\theta) \\ \quad - \alpha_t[x_t \sin\theta + y_t \cos\theta] - \omega_t^{\,2}(x_t \cos\theta - y_t \cdot \sin\theta) \tag{8}$$

$$\ddot{y}_p = \ddot{x}_t \sin\theta + \dot{x}_t \cos\theta \cdot \omega_t + \ddot{y}_t \cos\theta + \dot{y}_t(-\sin\theta)\cdot\omega_t + \dot{\omega}_t[x_t \cos\theta - y_t \sin\theta] \\ \quad + \omega_t[\dot{x}_t \cos\theta + x_t(-\sin\theta)\cdot\omega_t - \dot{y}_t \sin\theta - y_t \cos\theta \cdot \omega_t] \\ = \ddot{x}_t \sin\theta + \ddot{y}_t \cos\theta + \alpha_t[x_t \cos\theta - y_t \sin\theta] + 2\omega_t[\dot{x}_t \cos\theta - \dot{y}_t \sin\theta] \\ \quad - \omega_t^{\,2}[x_t \cdot \sin\theta + y_t \cdot \cos\theta] \tag{9}$$

By taking $dy + u = x_t \sin\theta + y_t \cos\theta$ and differentiate it with respect to θ, give

$$0 + \dot{u} = \dot{x}_t \sin\theta + x_t \cos\theta \cdot \omega_t + \dot{y}_t \cos\theta - y_t \sin\theta \cdot \omega_t \\ = \dot{x}_t \sin\theta + \dot{y}_t \cos\theta + \omega_t(x_t \cos\theta - y_t \sin\theta)$$

$$\dot{x}_t \sin\theta + \dot{y}_t \cos\theta = \dot{u} - \omega_t(x_t \cos\theta - y_t \sin\theta)$$

Substitute into equation (6), the kincmatic equation in x-component is

$$\ddot{x}_p = \ddot{x}_t \cos\theta - \ddot{y}_t \sin\theta - 2\omega_t[\dot{u} - \omega_t(x_t \cos\theta - y_t \sin\theta)] - \alpha_t(dy+u) - \omega_t^{\,2}(x_t \cos\theta - y_t \sin\theta) \\ = \ddot{x}_t \cos\theta - \ddot{y}_t \sin\theta - \alpha_t(dy+u) - 2\omega_t \cdot \dot{u} + \omega_t^{\,2} \cdot dx$$

with $dx = x_t \cos\theta - y_t \sin\theta$. (10)

Differentiate $dx = x_t \cos\theta - y_t \sin\theta$ with respect to θ, then

$$0 = \dot{x}_t \cos\theta - x_t \sin\theta \cdot \omega_t - (\dot{y}_t \sin\theta + y_t \cos\theta \cdot \omega_t) \\ = \dot{x}_t \cos\theta - \dot{y}_t \sin\theta - \omega_t(x_t \sin\theta + y_t \cos\theta)$$

$$\dot{x}_t \cos\theta - \dot{y}_t \sin\theta = \omega_t(x_t \sin\theta + y_t \cos\theta)$$

Substitute into (7), the kinematic equation in y-component can be written as

$$\ddot{y}_p = \ddot{x}_t \sin\theta + \ddot{y}_t \cos\theta + \alpha_t[x_t \cos\theta - y_t \sin\theta] + 2\omega_t[\dot{x}_t \cos\theta - \dot{y}_t \sin\theta] \\ \quad - \omega_t^{\,2}[x_t \sin\theta + y_t \cos\theta] \\ = \ddot{x}_t \sin\theta + \ddot{y}_t \cos\theta + \alpha_t \cdot dx + 2\omega_t[\omega_t(x_t \sin\theta + y_t \cos\theta)] - \omega_t^{\,2}(x_t \sin\theta + y_t \cos\theta) \\ = \ddot{x}_t \sin\theta + \ddot{y}_t \cos\theta + \alpha_t \cdot dx + \omega_t^{\,2}(x_t \sin\theta + y_t \cos\theta) \\ = \ddot{x}_t \sin\theta + \ddot{y}_t \cos\theta + \alpha_t \cdot dx + \omega_t^{\,2}(dy+u) \tag{11}$$

Since the backpack is model as a rigid body that moves with the trunk, then

$$\alpha_p = \alpha_t \tag{12}$$

Equations (1), (2), (3), (10) and (11) consist of 5 equations with 5 unknowns which are $\ddot{x}_p, \ddot{y}_p, \alpha_t, F_{xp}, F_{yp}$ and can be written in a matrix form

$$\begin{bmatrix} m_p & 0 & 0 & -1 & 0 \\ 0 & m_p & 0 & 0 & -1 \\ 0 & 0 & I_p & -(dy+u) & -dx \\ 1 & 0 & -(dy+u) & 0 & 0 \\ 0 & 1 & -dx & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x}_p \\ \ddot{y}_p \\ \alpha_t \\ F_{xp} \\ F_{yp} \end{bmatrix} = \begin{bmatrix} mg\sin\theta \\ -mg\cos\theta \\ M_{zp} \\ \ddot{x}_t\cos\theta - \ddot{y}_t\sin\theta - 2\omega_t\cdot\dot{u} + \omega_t^2\cdot dx \\ \ddot{x}_t\sin\theta + \ddot{y}_t\cos\theta + \omega_t^2(dy+u) \end{bmatrix}$$

By substituting equations (10) and (11) into (1) and (2), the matrix can be simplified into a 3x3 matrix which can be used to evaluate the forces (F_{xp} and F_{yp}) acting on the backpack in the x and y-components.

$$\begin{bmatrix} I_p & -(dy+u) & -dx \\ m_p(dy+u) & 1 & 0 \\ -m_p(dx) & 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha_t \\ F_{xp} \\ F_{yp} \end{bmatrix} = \begin{bmatrix} M_{zp} \\ m_p\left[\ddot{x}_t\cos\theta - \ddot{y}_t\sin\theta - 2\omega_t\cdot\dot{u} + \omega_t^2\cdot dx - g\sin\theta\right] \\ m_p\left[\ddot{x}_t\sin\theta + \ddot{y}_t\cos\theta + \omega_t^2(dy+u) + g\cos\theta\right] \end{bmatrix}$$

CONCLUSION

This paper presented a mathematical model of a human torso carrying a backpack in a sagittal plane. A Newton-Euler equation is applied to describe the backpack kinematics. From the developed model, the calculation for the forces and moments can be determined using an inverse dynamics method for variation of parameters.

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